OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

4728/01

MATHEMATICS

Mechanics 1

WEDNESDAY 10 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

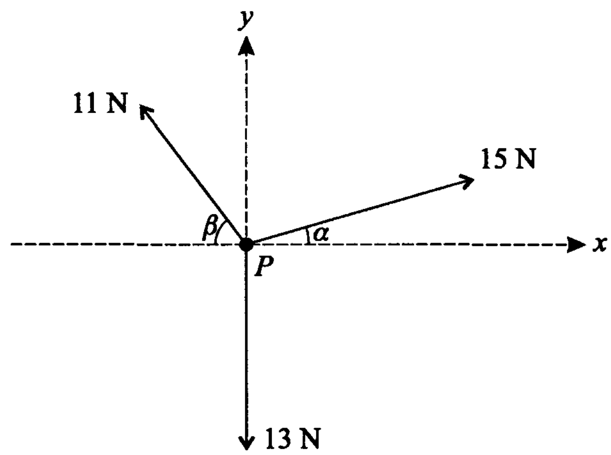
This document consists of **4** printed pages.

1 A trailer of mass 600 kg is attached to a car of mass 1100 kg by a light rigid horizontal tow-bar. The car and trailer are travelling along a horizontal straight road with acceleration $0.8\ \mathrm{m\,s^{-2}}$.

(i) Given that the force exerted on the trailer by the tow-bar is 700 N, find the resistance to motion of the trailer. [4]

(ii) Given also that the driving force of the car is 2100 N, find the resistance to motion of the car. [3]

2



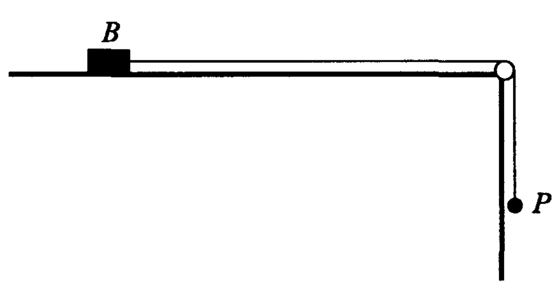
Three horizontal forces of magnitudes 15 N, 11 N and 13 N act on a particle P in the directions shown in the diagram. The angles α and β are such that $\sin\alpha = 0.28$, $\cos\alpha = 0.96$, $\sin\beta = 0.8$ and $\cos\beta = 0.6$.

(i) Show that the component, in the y-direction, of the resultant of the three forces is zero. [4]

(ii) Find the magnitude of the resultant of the three forces. [3]

(iii) State the direction of the resultant of the three forces. [1]

3

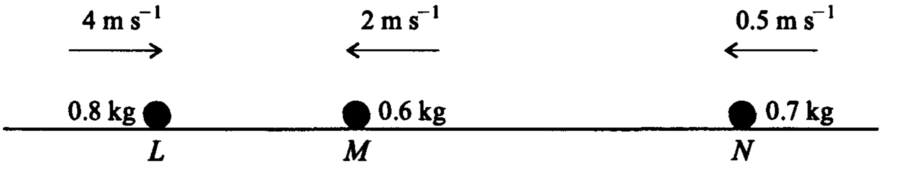


A block B of mass 0.4 kg and a particle P of mass 0.3 kg are connected by a light inextensible string. The string passes over a smooth pulley at the edge of a rough horizontal table. B is in contact with the table and the part of the string between B and the pulley is horizontal. P hangs freely below the pulley (see diagram).

(i) The system is in limiting equilibrium with the string taut and P on the point of moving downwards. Find the coefficient of friction between B and the table. [5]

(ii) A horizontal force of magnitude X N, acting directly away from the pulley, is now applied to B. The system is again in limiting equilibrium with the string taut, and with P now on the point of moving **upwards**. Find the value of X. [3]

4



Three uniform spheres L, M and N have masses 0.8 kg, 0.6 kg and 0.7 kg respectively. The spheres are moving in a straight line on a smooth horizontal table, with M between L and N. The sphere L is moving towards M with speed $4\,\text{m}\,\text{s}^{-1}$ and the spheres M and N are moving towards L with speeds $2\,\text{m}\,\text{s}^{-1}$ and $0.5\,\text{m}\,\text{s}^{-1}$ respectively (see diagram).

(i) L collides with M. As a result of this collision the direction of motion of M is reversed, and its speed remains $2\,\text{m}\,\text{s}^{-1}$. Find the speed of L after the collision. [4]

(ii) M then collides with N.

(a) Find the total momentum of M and N in the direction of M's motion before this collision takes place, and deduce that the direction of motion of N is reversed as a result of this collision. [4]

(b) Given that M is at rest immediately after this collision, find the speed of N immediately after this collision. [2]

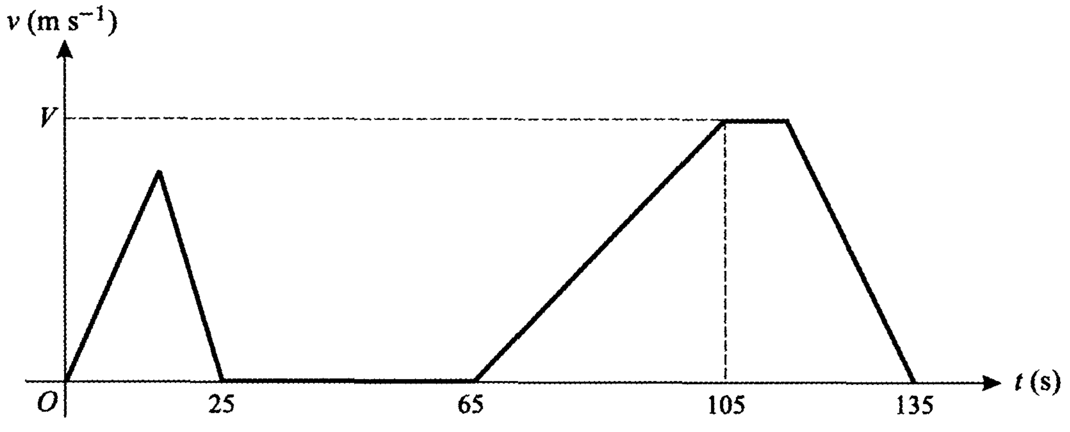
5 A particle starts from rest at a point A at time $t = 0$, where t is in seconds. The particle moves in a straight line. For $0 \leqslant t \leqslant 4$ the acceleration is $1.8t\ \text{m}\,\text{s}^{-2}$, and for $4 \leqslant t \leqslant 7$ the particle has constant acceleration $7.2\,\text{m}\,\text{s}^{-2}$.

(i) Find an expression for the velocity of the particle in terms of t, valid for $0 \leqslant t \leqslant 4$. [3]

(ii) Show that the displacement of the particle from A is 19.2 m when $t = 4$. [4]

(iii) Find the displacement of the particle from A when $t = 7$. [5]

[Questions 6 and 7 are printed overleaf.]



The diagram shows the (t, v) graph for the motion of a hoist used to deliver materials to different levels at a building site. The hoist moves vertically. The graph consists of straight line segments. In the first stage the hoist travels upwards from ground level for 25 s, coming to rest 8 m above ground level.

(i) Find the greatest speed reached by the hoist during this stage. [2]

The second stage consists of a 40 s wait at the level reached during the first stage. In the third stage the hoist continues upwards until it comes to rest 40 m above ground level, arriving 135 s after leaving ground level. The hoist accelerates at 0.02 m s^{-2} for the first 40 s of the third stage, reaching a speed of $V \text{ m s}^{-1}$. Find

(ii) the value of V, [3]

(iii) the length of time during the third stage for which the hoist is moving at constant speed, [4]

(iv) the deceleration of the hoist in the final part of the third stage. [3]

7 A particle P of mass 0.5 kg moves upwards along a line of greatest slope of a rough plane inclined at an angle of 40° to the horizontal. P reaches its highest point and then moves back down the plane. The coefficient of friction between P and the plane is 0.6.

(i) Show that the magnitude of the frictional force acting on P is 2.25 N, correct to 3 significant figures. [3]

(ii) Find the acceleration of P when it is moving

(a) up the plane,

(b) down the plane.

(iii) When P is moving up the plane, it passes through a point A with speed 4 m s^{-1}.

(a) Find the length of time before P reaches its highest point.

(b) Find the total length of time for P to travel from the point A to its highest point and to A.

1	**(i)**	Net force on trailer is +/-(700 - R_T)	B1		
			M1		For applying Newton's second law to the trailer with 2 terms on LHS (no vertical forces)
		700 - R_T = 600 x 0.8	A1ft		ft cv (+/-(700 - R_T))
		Resistance is 220N	A1	4	
	(ii)		M1		For applying Newton's second law to the car or to the whole, with a =+/- 0.8 (no vertical forces)
		2100 – 700 – R_C = 1100 x 0.8	A1ft		
		or 2100 – (R_C + 220) = (1100 + 600)x 0.8			ft cv(220)
		Resistance is 520N	A1	3	

2	**(i)**		M1		For resolving forces vertically
		15 x 0.28 and 11x 0.8	A1		Allow use of □ = 16.3 and □ =53.1
		Y= 15x0.28 + 11x0.8 - 13	A1ft		Ft cv(15 x 0.28 and 11x 0.8)
		Component is zero AG	A1	4	**SR** 15sin□ + 11sin□ -13 = 0 gets M1A0A1ftA0
	(ii)		M1		For resolving forces horizontally
		X = 15 x 0.96 – 11 x 0.6	A1		Allow use of □ = 16.3 and □ =53.1
		Magnitude is 7.8N	A1	3	Accept 7.79, -7.8
	(iii)	Direction is that of the (+ve) x -axis	B1	1	Do not allow horizontal, 90° from vertical. Do not award if □ = 16.3 and □ =53.1 have been used.

3	**(i)**	T = 0.3g	B1		At particle (or 0.3g -T= 0.3a)
		F = T	B1		Or F = cv(T at particle) (or T - F = 0.4a)
		R = 0.4g	B1		
			M1		For using F = μR
		Coefficient is 0.75	A1	5	
	(ii)		M1		For resolving 3 relevant forces on B horizontally, a=0
		X = 0.3g + 0.3g	A1ft		Ft X = 0.3g + cv(μ) cv(R)
		X = 5.88N	A1	3	

4	**(i)**	Momentum before collision = +/-(0.8 x 4 - 0.6 x 2)	B1		Or momentum change L $0.8x4 +/- 0.8v_L$ Accept inclusion of g in both terms
		Momentum after collision = $+/-0.8v_L + 0.6 x 2$	B1		Momentum change N 0.6x2 + 0.6x2 Accept inclusion of g in both terms
			M1		For using the principle of conservation of momentum even if g is included throughout
		Speed is 1 ms^{-1}	A1	4	Accept -1 from correct work (g not used).
	(ii)(a)	0.6x2 - 0.7x0.5	M1		Must be a difference. **SR** 0.6x1 - 0.7x0.5 M1
		Total is 0.85$kgms^{-1}$	A1		Must be positive
		<u>Total</u> momentum +ve after the collision. If N continues in its original direction, both particles have a negative momentum.	DM 1		Or 0.6v + 0.7w is positive, confirming that the momentum is shared between two particles. No reference need be made to the physically impossible scenario where M and N both might continue in their original directions.
		N must reverse its direction.	A1	4	
	(ii)(b)	0.6x2 - 0.7x0.5 (= 0.85) = 0.7v	A1ft		ft cv (0.85). Award M1 if not given in ii(a).
		Speed is 1.21ms^{-1}	A1	2	Positive. Accept (a.r.t) 1.2 from correct work

5	**(i)**	$1.8t^2/2$ (+C)	M*1		For using $v = \int a dt$
		(t = 0,v = 0) C = 0	B1		May be awarded in (ii). Accept c written and deleted.
		Expression is $1.8t^2/2$	A1	3	also for $1.8t^2$ +c
	(ii)		M1		For using $s = \int v dt$
		$0.9t^3/3$ (+K)	A1		**SR** Award B1 for (s = 0, t = 0) K = 0 if not already given in (i), or +K included and limits used.
		0.3 x 64	M1		For using limits 0 to 4 (or equivalent)
		19.2m AG	A1	4	
	(iii)	u = 0.9 x 4^2	D* M1		For using 'u' = v(4)
			M1		For using s = ut + ½ x$7.2t^2$ with non-zero u
		s = 14.4 x 3 + ½ 7.2 x 3^2	A1		(s = 75.6)
		19.2 + 75.6	M1		For adding distances for the two distinct stages
		Displacement is 94.8m	A1	5	
		OR			
		$v = \int 7.2 dt$	D* M1		For finding v(4) Integration and finding non-zero integration constant
		t = 0, v = 14.4, c = 14.4			Nb Using t=4, v=14.4 gives c = -14.4 $s = \int 7.2t - 14.4 dt$
		$s = \int 7.2t + 14.4 dt$			Integration and finding integration constant.
		t = 0, s = 0, k = 0			Nb t=4 with s=19.2 and v=7.2t-14.4 gives k=19.2
			M1		Substituting t = 3 (OR 7 into s = $3.6t^2$ - 14.4t + 19.2)
		s=3.6x3^2+14.4x3	A1		(s=75.6) (OR s = 3.6 x7^2 - 14.4x7 + 19.2)
		19.2 + 75.6 = 94.8	M1		Adding two distinct stages OR
		Displacement is 94.8m	A1		s = 3.6 x7^2 - 14.4x7+19.2 =94.8 final M1A1

6	**(i)**	½ $25v_m$ = 8 or ½Tv_m+½(25 – T)v_m =	B*1		Do not accept solution based on isosceles or right angled triangle

	8 Greatest speed is 0.64 ms^{-1}	D*B 1	2	
(ii)		M1		For using v = u + at or the idea that gradient represents acceleration
	V = 0.02 x 40	A1		
	V = 0.8	A1	3	
(iii)		M1		For using the idea that the area represents displacement. nb trapezium area is 16+8+8
		M1		For A = ½ (L_1 + L_2)h or other appropriate breakdown
	½ (70 + T) x 0.8 = 40 - 8	A1ft		½ (30 + T) x 0.8 = 40 - 8- ½ x 40 x 0.8 ft cv(0.8)
	Duration is 10s	A1	4	
(iv)		M1		For using v = u + at or the idea that gradient represents acceleration
	0=0.8+a(30–10)	A1ft		ft cv(10) and cv(0.8)
	Deceleration is 0.04ms^{-2}	A1	3	Accept -0.04 from correct work
	Or	M1		Using the idea that the area represents displacement.
	40-8-½ x 40 x 0.8-10x0.8 =0.8(30-10)-a(30–10)^2/2	A1ft		Ft cv(0.8 and 10)
	Deceleration is 0.04ms^{-2}	A1		Accept -0.04 from correct work. d=-0.04 A0

7	**(i)**	$R = 0.5g\cos 40^{\circ}$	B1		R = 3.7536
		$F = 0.6 \times 0.5g\cos 40^{\circ}$	M1		For using $F = \mu R$
		Magnitude is 2.25N AG	A1	3	
	(ii)		M1		For applying Newton's second law (either case) //slope, two forces
		$-/+0.5g\sin 40^{\circ} - F = 0.5a$	A1		Either case
		(a) Acceleration is -10.8ms^{-2}	A1		Accept 10.8 from correct working (both forces have the same sign)
		(b) Acceleration is 1.79ms^{-2}	A1	4	Accept -1.79 from correct working (the forces have opposite sign) Accept ! 1.8(0)
	(iii)a)	$0 = 4 + (-10.8)T_1$	M1		Requires appropriate sign
		$T_1 = 0.370(3)$	A1		Accept 0.37
	b)		M1		For complete method of finding distance from A to highest point using a(up) with appropriate sign
		$0 = 4^2 + 2(-10.8)s$ or $s = (0 + 4) \times 0.37/2$ or $s = 4(0.370) + ½(-10.8)(0.370)^2$	A1 ft		ft a(up) and/or T_1 (s = 0.7405)
			M1		For method of finding time taken from highest point to A and not using a(up)
		$0.7405 = ½(1.79)T_2^2$	A1ft		ft a(down) and cv(0.7405) (T_2 = 0.908 approx)
		0.370 + 0.908	M1		Using $T = T_1 + T_2$ with different values for T_1, T_2
		= 1.28s	A1	8	3 significant figures cao